Pressure Dependent Hydraulic Modelling for Water Distribution Systems under Abnormal Conditions

Zheng Yi Wu and Thomas Walski

Bentley Systems Inc, Haestad Methods Solution Center, 27 Siemon Co Dr., Suite200W, Watertown, CT06795, USA, zheng.wu@bentley.com

Abstract A water distribution system provides water services to customers under normal and abnormal conditions. It is important for water companies to be informed to what degree or level that a water system is able to supply its customers when an emergency or calamity scenario occurs. Such an abnormal event can cause one or more than one element out of service. When such an event occurs, pressure is often lower than the operating condition. Consequently the service can only be maintained to a certain level, the supplied demand is not as much as the expected before the outage is fully recovered. In this paper, a new approach is developed and presented to accurately simulate those abnormal events. Network hydraulic model is enhanced to simulate the actual demand according to the available pressure, so-called pressure dependent demand modelling, By applying the improved hydraulic modelling approach, engineers are able to analyze a variety of emergence conditions including the planned or unplanned system maintenance, outages of supply facilities (pipes, pumps and tanks) as well as disconnected customers. A number of examples are presented to demonstrate the application of the pressure dependent demand modelling approach.

Keywords Emergence operation, hydraulic model, pressure dependent demand and water supply systems

Introduction

A water distribution model is represented by using a link-node formulation that is governed by two conservation laws, namely mass balance at nodes and energy conservation along a hydraulic loop. The node is where water consumption is allocated and defined as demand, which is treated as a known value so that nodal hydraulic head can be solved. This formulation is valid only if the hydraulic pressures at all nodes are adequate so that the demand is independent of pressure. However, in many cases nodal pressure is not sufficient for supplying the desired demand. These cases may include the planned system maintenances, unplanned pipe outages, power failure at pump stations, and insufficient water supply from water sources.

In 1994, the Dutch water authority posted the guideline for water companies to evaluate the level of water supply while coping with calamity events. A tentative guideline requirement is that a water system must meet 75% of the original demand for the majority of customer and no large group of customers (2000 resident addresses) should receive less than 75% of their original demand. The guideline is applicable to all the elements between the source and tap. Dutch water companies are required to find the effect of every element in a water system. In order to calculate the water supply level under a calamity event, a hydraulic modelling approach is proposed:

1. Take one element at a time out of a model, copying the calamity event of element outage
2. Run the model for peak hours of all demand types and also the peak hours of tank filling. The actual demand needs to be modelled as a function of pressure, the supply is considered unaffected if the pressure is above the required pressure threshold
3. Evaluate the water supply level for each demand node. If there are less than 2000 resident customers receiving less than 75% of the normal demand, then the requirement is met. Repeat Step 1 to simulate another calamity event. If the requirement is not met, continue with step 4.
4. Perform 24 hours pressure dependent demand simulation for the maximum demand day under the calamity even
5. Sum up the actual demand for each node over 24 hours
6. Check if there is any node where the totalized demand over 24 hours is less than 75% of the maximum day demand, if not the guideline is met. Otherwise an appropriate system improvement needs to be identified in order to meet the guideline.

UK water companies are also required by regulation to provide water at a pressure that will, under abnormal circumstances, enable it to reach the top floor of a house. In order to assess if this requirement is satisfied, companies are required to report against a service level corresponding to a pressure head of 10 meters at a flow of 9 litres per minute. In addition, water companies are expected to report the supply reference for unplanned and planned service interruptions.

Both use cases provide some generality for water utilities world wide to evaluate the performance of water systems under emergency and low pressure conditions. An emergency event can be specified as one set of element outages. In order to quantify the water supply level under such an event the demand must be modelled as a function of nodal pressure. Hydraulic model needs to be enhanced to perform pressure dependent demand simulation and to compute the level of water supply under variety of facility outages.

Pressure Dependent Demand

Whenever a calamity occurs, the systems pressures are affected. Some locations may not have the required pressure. Nodal demand, the water available at a location, is certainly dependent on the pressure at the node when the pressure is low. In other words, unlike the conventional approach of demand-driven analysis, demand is a function of pressure, so-called pressure dependent demand (PDD), however, it is believed that a junction demand is not affected by pressure if the pressure is above a threshold. The junction demand is reduced when the pressure is dropping below the pressure threshold and it is zero when the pressure is zero.

PDD can be defined as one of two pressure-demand relationships including a power function and a pressure-demand piecewise linear curve (table). The power function is given as:

$$\frac{Q_i^s}{Q_{ri}} = \begin{cases} 0 & H_i \le 0 \\ \left(\frac{H_i}{H_{ri}}\right)^{\alpha} & 0 < H_i < H_t \\ \left(\frac{H_t}{H_{ri}}\right)^{\alpha} & H_i \ge H_t \end{cases} \quad (1)$$

Where H_i represents the calculated pressure at node i; Q_{ri} denotes the requested demand or reference demand at node i; Q_i^s is the calculated demand at node i; H_{ri} designates the reference pressure that is deemed to supply full requested/reference demand; H_t is the pressure threshold above which the demand is independent of nodal pressure and α is the exponent of pressure demand relationship.

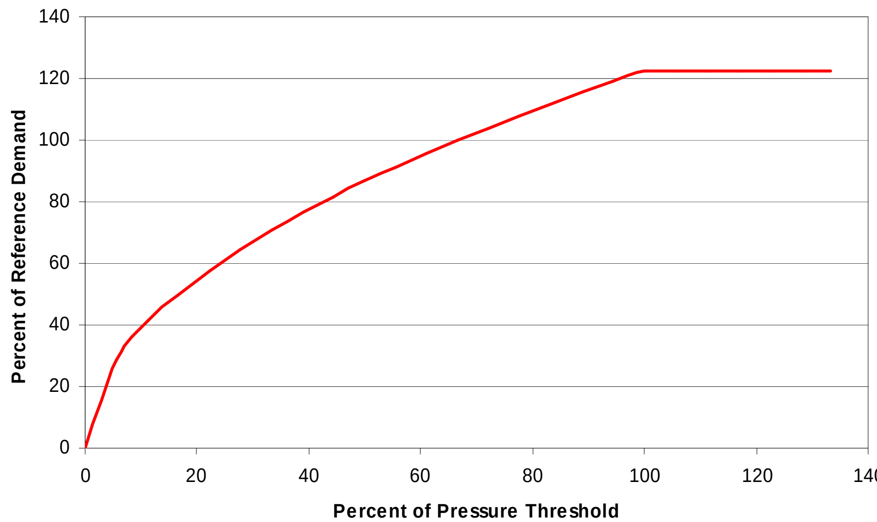

Figure 1 A Typical Pressure Dependent Demand Curve

A typical PDD power function is illustrated in the Fig. 1. The actual demand increases to the full requested demand (100%) as pressure increases, but remains constant after the pressure is greater than the pressure threshold, namely the percent of pressure threshold is greater than 100%. Alternatively, PDD function can be specified as a pressure-demand piecewise linear curve or a table of pressure percentage vs demand percentage. Pressure percentage is the ratio of actual pressure to a nodal threshold pressure while demand percentage is the ratio of the calculated demand to the reference demand.

Unlike the conventional water distribution model where the nodal demand is a known value, pressure dependent demand modelling approach stipulates that both the nodal demand

and pressure are unknown. Solving for such a hydraulic model requires for reformulating the solution method as follows.

Solution Methodology

The key solution methodology is how to solve for the pressure dependent demand as given by Eq.(1). Conventionally, nodal demand is a known value. Applying the mass conservation law to each node and energy conservation law to each loop, the network hydraulics solution can be obtained by iteratively solving a set of linear and quasi-nonlinear equations. A unified formulation for solving network hydraulics is given as global gradient algorithm (GGA) as (Todini 1988):

$$\begin{bmatrix} A_{11} & \dots & A_{12} \\ \dots & \dots & \dots \\ A_{21} & \dots & 0 \end{bmatrix} \begin{bmatrix} Q \\ \dots \\ H \end{bmatrix} = \begin{bmatrix} -A_{10}H_0 \\ \dots \\ -q \end{bmatrix} \tag{2}$$

Where Q is the unknown pipe discharge and H is the unknown nodal head. q is the set of nodal demand that is not dependent on the nodal head H.

For pressure dependent demand, the demand is no longer a known value, but a function of nodal pressure (head) as defined in Eq(1). The solution matrix (2) becomes:

$$\begin{bmatrix} A_{11} & \dots & A_{12} \\ \dots & \dots & \dots \\ A_{21} & \dots & A_{22} \end{bmatrix} \begin{bmatrix} Q \\ \dots \\ H \end{bmatrix} = \begin{bmatrix} -A_{10}H_0 \\ \dots \\ -q \end{bmatrix} \tag{3}$$

Now, a new diagonal matrix A_{22} is added to the solution matrix (3). The non-zero diagonal element is given as:

$$A_{22}(i,i) = Q_i^s \tag{4}$$

It depends on which supply characteristic equation Eq.(1) is applied. There are two approaches to solve the pressure dependent demand as formulated in Eq. (3). By following the original derivation of GGA, pressure dependent demand formulation can be solved as:

$$\begin{bmatrix} D_{11} & \dots & A_{12} \\ \dots & \dots & \dots \\ A_{21} & \dots & D_{22} \end{bmatrix} \begin{bmatrix} dQ \\ \dots \\ dH \end{bmatrix} = \begin{bmatrix} dE \\ \dots \\ dq \end{bmatrix} \tag{5}$$

The only difference from the original GGA is the new diagonal matrix D_{22}, which is the deviation of A_{22} of pressure head H. For supply characteristic defined as Eq.(1), the corresponding expression is

$$D_{22}(i,i) = \begin{cases} 0 & H_i \le 0 \\ \alpha \left(\dfrac{H_i}{H_t} \right)^{\alpha - 1} \times Q_i & 0 < H_i < H_t \\ 0 & H_i \ge H_t \end{cases} \tag{6}$$

The modified GGA is to calculate D_{22} for each pressure dependent demand node and add to $A(i, i)$ as follows.

$$A(i,i) = \sum_j p_{ij} - D_{22}(i,i) \tag{7}$$

Where j denotes the pipe j that is connected with node i. This notation is the same as EPANET formulation (Rossman 1994).

Results

Pressure dependent demand model formulated above is implemented as one of the modelling functions in WaterCAD and WaterGEMS (Bentley 2005), along with a set of alternative data specification for conducting pressure dependent demand calculation. This is one of the latest modelling feature and a relatively new way of modelling water system for under practical conditions. Modelling examples are presented to ensure the accuracy of the new modelling method, the integrity and consistency with the existing modelling options and also to illustrate the possible applications of the approach.

Example I

This is a trivial example model, composed by one reservoir, one pipe and one node as in Fig. 2. The main purpose is to verify the accuracy and robustness of the PDD calculation at different levels of nodal pressures. A demand of 10 cfs is specified at node J-1, along with the reservoir elevation of 100 meters. A range of node elevations, from 0 to 110.0 meters, are applied to test the PDD calculation. A pressure threshold of 30 meters (92 ft) and exponent of 0.5 have been used for testing.

Figure 2(a), (b) and (c) illustrate the calculated PDD at different nodal pressure for this example. When the elevation at J-2 is specified below 20 meters, the pressure is found above the threshold 92 ft, so that the actual PDD is equal to the demand of 10 cfs as shown as in Fig 2(a). As the nodal elevation at J-2 increases, the pressure decreases and so does the calculated demand. Figure 2(b) shows that the demand is less than the required demand of 10 cfs when the pressure is below the threshold pressure of 92 ft. The demand becomes zero when the elevation increase to above 100 meters and the pressure is zero. The calculated nodal demand (calculated PDD) and the specified demand (specified PDD) by the PDD function are compared for a large number of variations of nodal pressures at J-2 for this example. The comparison is as shown as in Fig.3. It demonstrates that the calculated demand matches well with the prescribed PDD function for this example.

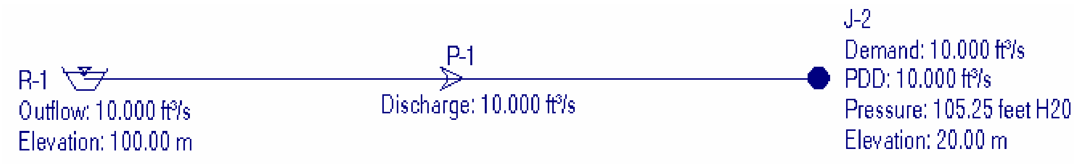

Figure 2(a) Calculated PDD is the same as demand due to the pressure is above the pressure threshold.

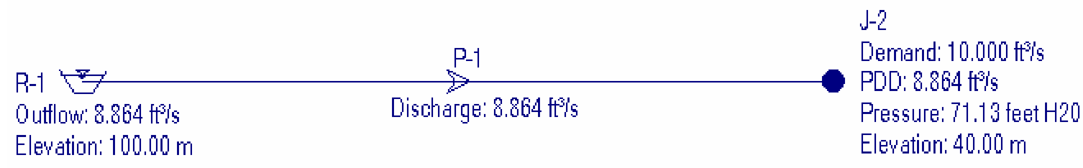

Figure 2(b) Calculated PDD is less than the required demand due to the pressure is lower than the pressure threshold.

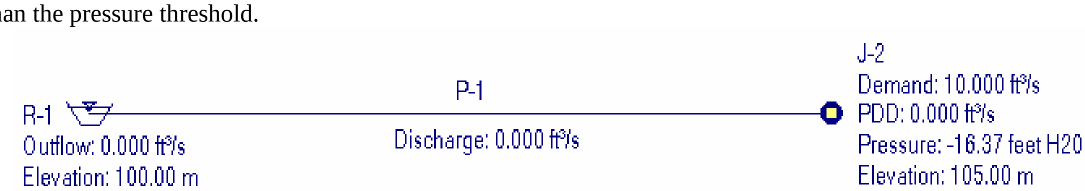

Figure 2(c) Calculated PDD is zero due to the pressure is negative.

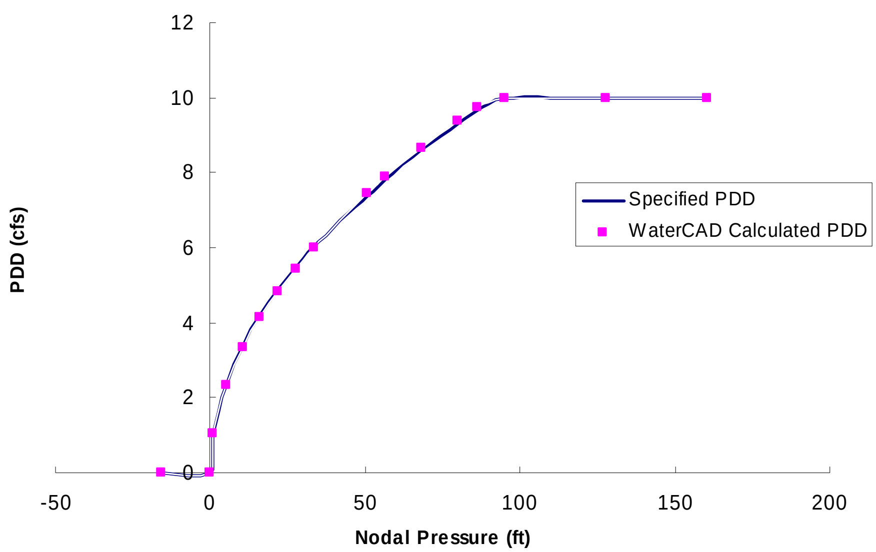

Figure 3 Comparison of the calculated PDD with specified PDD for exponent of 0.5

Example 2

This example is composed of one reservoir and 7 pipes that form two loops as shown in Fig. 4. The model is set up with base demand of 2 cfs (56.62 L/s) at node J-2 and J-6, and 1 cfs (28.32 L/s) at node J-3, J-4 and J-5. It is assumed that the required demand is supplied with the sufficient nodal reference pressure of 40 meters. Both conventional model and pressure dependent demand model are applied to analyzed two operating scenarios of pipe outages as follows.

- Scenario I: Pipe P-7, one of two pipes connecting to the water source, is out of service.
- Scenario II: Node J-6 is isolated from the system due to the planned maintenance of pipe P-5 and P-7.

The modelling results are summarized and compared as in Table 1 for all five model runs including three conventional model (fixed demand method) runs of normal and abnormal conditions and two pressure dependent demand model runs of abnormal conditions. Under normal conditions, nodal demand is supplied with the pressures greater than 40 meters. The results obtained by using the conventional modelling approach indicate that the nodal demand is met regardless of the pressure drop for abnormal conditions when pipe P-7 is shut down. For J-6 is isolated from the system, the simulated results by conventional approach are misleading that the isolated node J-6 is still supplied with the required demand of 56.62 L/s but the pressure is a huge negative value at J-6. In comparison, pressure dependent demand approach show more practically meaningful results for both abnormal conditions. The available demand is reduced as pressure decreases when pipe P-7 is outage.

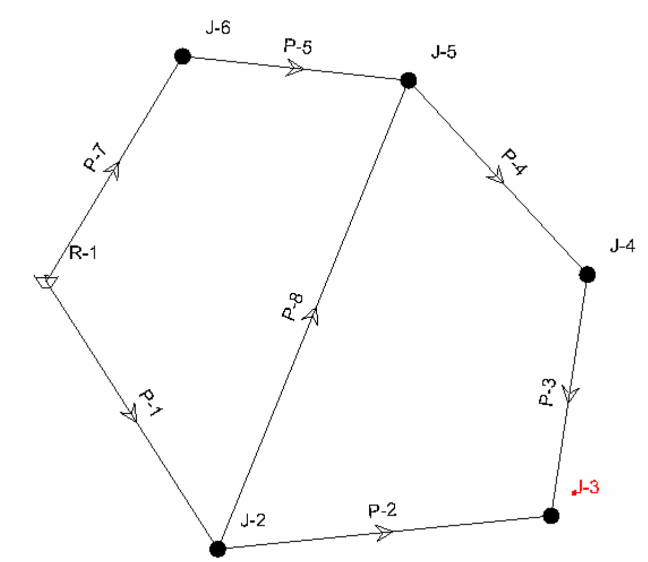

Figure 4 A sample system with the nodal reference pressure head of 40 meters and threshold pressure of 65.0 meters

Table 1 Comparison of the simulated results by conventional and PDD modelling approach for abnormal conditions

	Normal Conditions		Abnormal Conditions							
			Pipe P-7 Outage				Node J-6 Isolated			
	Conventional Model Results		Conventional Model Results		PDD Model Results		Conventional Model Results		PDD Model Results	
Node	Q (L/s)	P (m)	Q (L/s)	P (m)	Q (L/s)	P (m)	Q (L/s)	P (m)	Q (L/s)	P (m)
J-2	56.63	43.40	56.63	23.49	49.74	30.73	56.63	30.05	54.56	36.97
J-3	28.32	42.80	28.32	19.94	23.84	28.24	28.32	27.59	26.77	35.60
J-4	28.32	42.80	28.32	18.90	23.53	27.52	28.32	27.05	26.70	35.43
J-5	28.32	43.30	28.32	18.72	23.49	27.40	28.32	27.04	26.73	35.49
J-6	56.63	44.43	56.63	17.23	46.09	26.38	56.63	-30418567.33	0.00	0.00

Example III

This example is designed to evaluate the impact of a typical emergency operation condition that a large water user, such as for fulfilling fire fighting requirement, may constantly pump water out of the system. Such an event often results in a low pressure on the surrounding customers. Water service may not be provided as expected under normal condition. Pressure dependent demand approach is demonstrated on the analysis of the impact by a large water user. The example system, as shown as in Figure 5, originally used for studying the Chlorine decay kinetics (Vasconcelos et al. 1997) and optimizing water quality model calibration (Wu 2006), has been used for applying pressure dependent demand approach to simulate the large water user impact on the surrounding customers.

It is assuming that a large water demand of 450 gpm at node OH24 is constantly required for an emergency purpose (such as fire fighting). The scenario is simulated by using the conventionally fixed demand modelling and pressure dependent demand approach. Figure 6 illustrates that the large demand can be satisfied most of time with the minimum pressure requirement of 20 psi. The demand impact on the other customers at node OH28 is exemplified as Fig. 7. The conventional model shows that the required demand is met regardless pressure drop caused by the large demand at node OH24. In comparison, pressure dependent approach shows that the available demand is less than the required demand due to the reduced pressure by the large water consumption at node OH24.

Conclusions

Water distribution model under abnormal conditions requires for enhancing the hydraulic network model to handle pressure dependent demand. The conventional approach is not appropriate for correctly modelling pressure dependent demand scenario. This paper successfully developed and implemented the pressure dependent demand model. Examples presented in this paper demonstrate that the enhanced modelling approach enables practical engineers to accurately analyze many abnormal conditions including water supply level evaluation, element criticality analysis, pipe outages, disconnected customers and emergency operation conditions. Equipped with the new modelling function, engineers are capable of evaluating the system performance and deficiency, and thus ultimately identifying the remediation for water systems to comply the regulations and guideline.

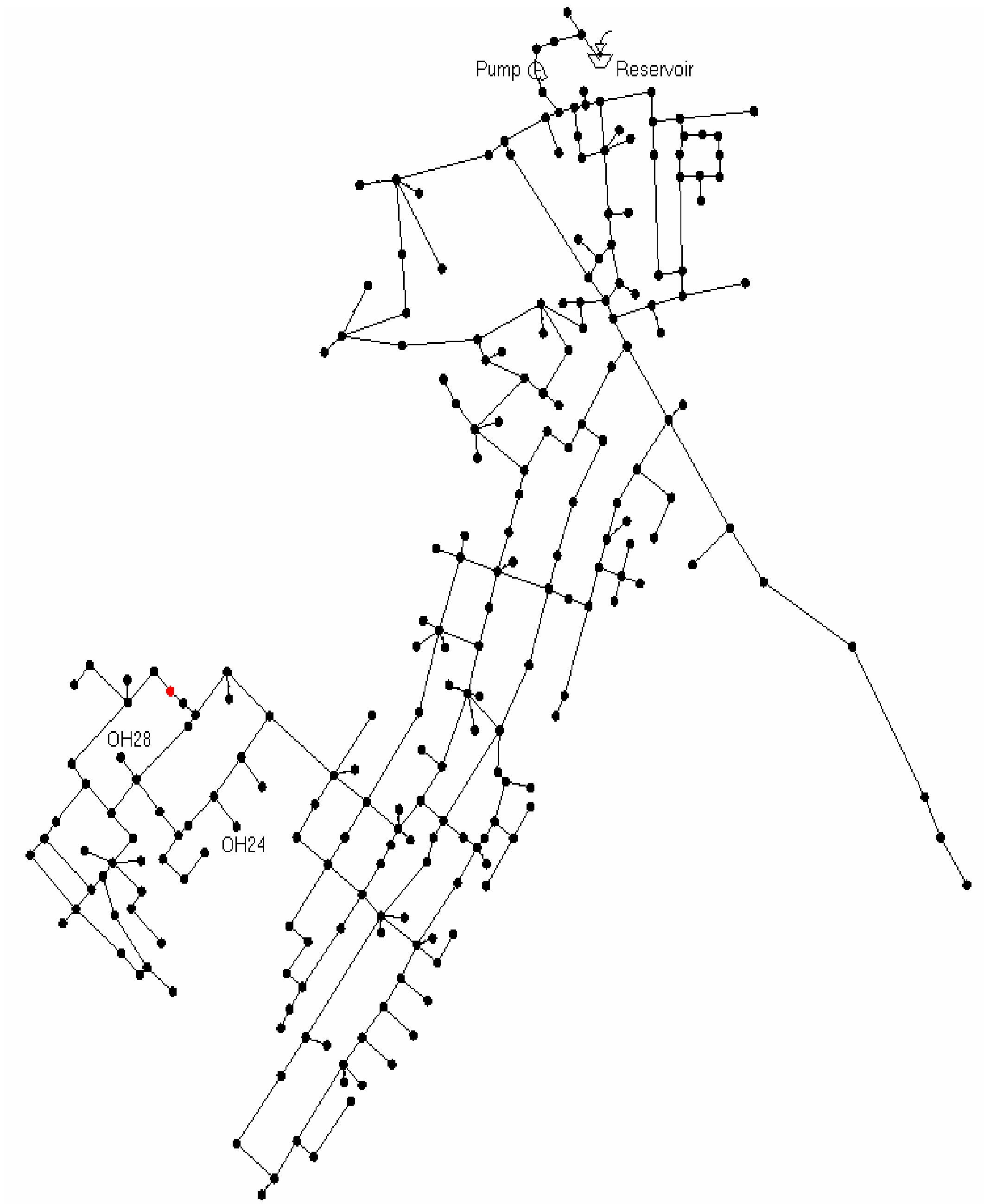

Figure 5 Layout of Oberlin Zone in Harrisburg System

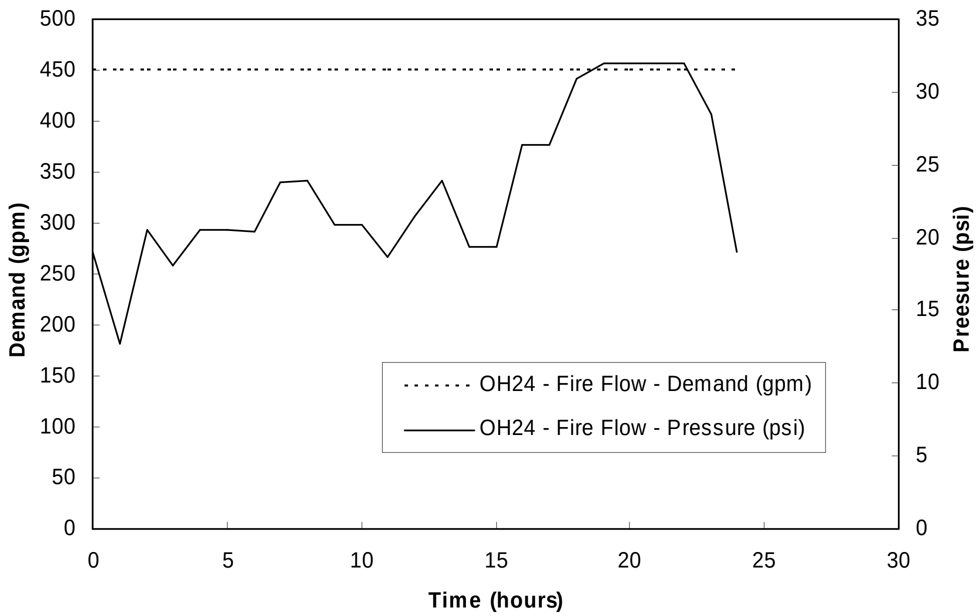

Figure 6 Calculated pressure and demand when a big user constantly draining water at node OH24 out of the system

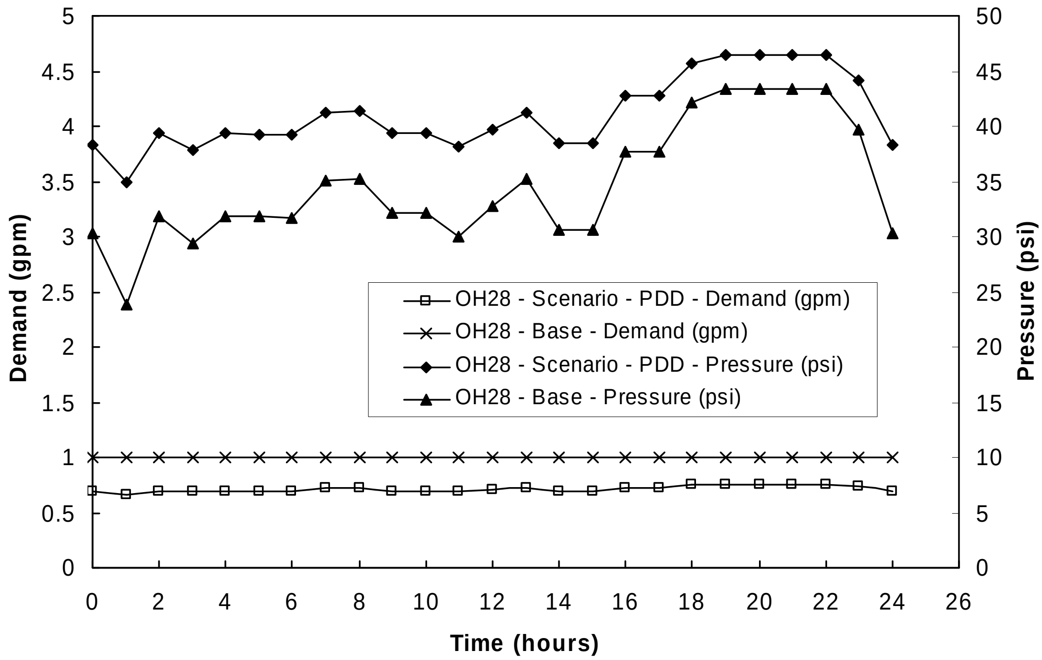

Figure 7 Comparison of the simulated demand by conventional model and pressure dependent demand model when emergence operation occurs at OH24

References

Bentley Systems, Incorporated, (2005). *WaterGEMS User Manual.* 27 Siemon Co Dr, Suite200W, Watertown, CT06795, USA.

Rossman, L.A. (1994). *EPANET Users Manual.* Drinking Water Research Division, Risk Reduction Engineering Laboratory, Office of Research and Development, U.S. Environmental Protection Agency, Cincinnati, Ohio. USA.

Todini, E. & Pilati, S. (1988). A gradient algorithm for the analysis of pipe network. In Coulbeck B. and Chun-Hou O. (eds). *Computer Applications in Water Supply, Vol. 1 – System Analysis and Simulation,* John Wiley & Sons, London, pp.1-20.

Vasconcelos, J. J.; Rossman, L. A.; Grayman, W. M.; Boulos, P. F. & Clark R. M. Kinetics of Chlorine Decay, **1997**, J. of AWWA, *89*,7.

Wu, Z. Y. (2006) "Optimal Calibration Method for Water Distribution Water Quality Model." *Journal of Environmental Science and Health (A),* June Issue.