WATER DISTRIBUTION SYSTEM ANALYSIS BEFORE DIGITAL COMPUTERS

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Abstract

Water distribution system analysis did not begin with the development of digital computer programs. Engineers were successfully designing, constructing and operating water distribution systems long before the coming of the computer age.

This paper traces the development of analysis from Archimedes and the Roman aqueducts, through the development of principles of fluid flow by Newton, Bernoulli and Euler to the development of head loss equations by Chezy, Darcy, Weisbach, Hazen, Williams and Moody. It then looks at how principles developed for individual elements where combined to solve network problems by Cross and the subsequent development of analog computer methods.

The paper will show how our understanding of hydraulics did not come about quickly but through an unraveling of one problem after another by some brilliant individuals.

EARLY DAYS

Today, engineers take for granted the fact that they can analyze complex pipe networks at the click of a mouse button. However, most major water systems around the world were constructed long before hydraulic analysis models were available. A great deal was accomplished with only graphs, slide rules and good judgment. In the earliest days, even slide rules weren't available.

In early history, there was no formal field of hydraulics. Numerous scientists contributed to the understanding of fluid flow. One of the most notable in ancient times was Archimedes, who reported on the principles of buoyancy and is credited with developing the screw pump, although his pump was a refinement of earlier designs (Rouse and Ince, 1980). Hero's book, *DioptraI,* is the earliest expression of the relationship of velocity, area and flow (Rouse and Ince, 1980).

Sextus Julius Frontinus is credited with writing the first hydraulics books in 97 AD, describing the construction of the Rome water supply system (Mays, 2000). Roman engineering was based more on rules of thumb rather than scientific principles. Early water systems relied primarily on open channel flow.

The science of hydraulics did not advance much until the time of Leonardo da Vinci when his treatise *Del moto e misura dell'acqua* summarized the state of the art of hydraulics circa 1500 (Rouse and Ince, 1980).

PIPING HYDRAULICS

By the early in the 17th century Benedetto Castelli had formulated the relationship between velocity, flow and conduit size.

$Q = AV$

Where Q = flow
A = flow area
V = average velocity

His student Evangelista Torricelli showed that there is a relationship between the velocity of a fluid and the square root of the head loss (Rouse and Ince, 1980).

Isaac Newton in the early 18th century developed the fundamental laws of motion which serve as the basis for subsequent understanding of hydraulics. In addition to his laws of motion, Newton's Law of Viscosity, which states that shear resistance is proportional to the velocity gradient.

$$\tau = \mu \frac{du}{dy}$$

Where τ= shear stress
μ = viscosity
u = point velocity
y = distance from boundary

Daniel Bernoulli and his father Johann (mid 18th century) developed many of the principles for analyzing fluid flow and Daniel is credited with publishing the book *Hydrodynamica* which was the most thorough hydraulics book of its time. The equation attributed to Bernoulli and many of the other basic equations in hydraulics were actually developed by Leonhard Euler in the mid 1700s (Rouse and Ince, 1980). Along any streamline

$$z_1 + \frac{v_1^2}{2g} + \frac{p_1}{\gamma} = const$$

For pipe flow, this could be extended to include energy loss which meant that between any two points,

$$z_1 + \frac{v_1^2}{2g} + \frac{p_1}{\gamma} = z_2 + \frac{v_2^2}{2g} + \frac{p_2}{\gamma} + \Delta h$$

Where z = elevation
v = velocity
g = acceleration due to gravity
p = pressure
γ = specific weight of fluid
Δh = change in head due to energy loss or pumping (negative)

Henri de Pitot showed that the velocity of a fluid is proportional to the square root of the head in the early 1700s. The equation was very useful for open channel flow but it wasn't until centuries later that it was successfully used for closed pipes and hydrant discharge. This principle has served as the basis for Pitot flow meters used to this day.

$$v = \sqrt{2g \; \Delta h}$$

While investigators realized that it took energy to move fluids, Antoine Chezy was the first to extend this idea to show that head loss in a fluid is proportional to the velocity squared. All subsequent head loss equations in turbulent flow are related back to his work (Walski et al., 2003).

$$v = C\sqrt{RS}$$

Where C = Chezy's constant
R = hydraulic radius
S = energy slope

By 1840, Gotthilf Hagen and Jean Louis Poiseuille were able to develop an analytical equation for predicting head loss in laminar flow. While laminar flow is not of much significance in water distributions, this equation is part of the overall understanding of head loss in pipes. While Hagen was a hydraulic engineer, Poiseuille was actually a medical doctor interested in blood flow (Rouse and Ince, 1980).

$$v = \frac{\Delta p D^2}{32 \mu L}$$

Where D = pipe diameter
L = length

This work and Chezy's equation was extended to a more general formula by Julius Weisbach and Henry Darcy circa 1845 (Rouse and Ince, 1980). Their work was similar to earlier work of Prony. The equation has the common form given below and is still the most theoretically correct formula for head loss in pipe flow.

$$\Delta h = f \, \frac{L v^2}{2gD}$$

Where f = dimensionless friction factor

Difficulty in predicting f for practical problems made use of this equation more troublesome than it is today, which lead researchers to develop easier but less correct methods. The Darcy-Weisbach equation has had several names including Weisbach equation, Darcy equation and Fanning equation but Rouse is credited with standardizing the name in 1962 (Brown, 2003).

Other head loss equations, more applicable to open channel rather than closed pipe flow, were developed by Henri Bazin and Wilhelm Kutter in the early 19th century and Robert Manning in the late 19th century (Rouse and Ince, 1980). These tended to be more oriented to open channel flow.

In 1883, Osborne Reynolds investigated the different flow regimes and was able to clearly define the distinction between laminar and turbulent flow and identified the dimensionless number which is used to characterize the different types of flow.

$$R = \frac{VD\rho}{\mu}$$

While the Darcy-Weisbach equation could be used to determine head loss in pipes, determining the friction factor was difficult. Alan Hazen and G. S. Williams (1906) developed an equation for head loss in rough turbulent flow with a C factor instead of the friction factor. The C-factor is significantly more constant and easier to use and Williams and Hazen published easy to use tables to facilitate the calculations, which lead to the widespread use of the Hazen-Williams equation and to this date is widely used in engineering practice even though it is less accurate than the Darcy-Weisbach equation over a wide range of flows.

$$h = \frac{6.78L}{D^{1.167}}\left(\frac{v}{C}\right)^{1.852}$$

Where C = Hazen-Williams C
The 6.78 becomes 3.02 in English units

In spite of its widespread use, the Hazen-Williams equation is looked upon with contempt by academics. Rouce and Ince (1980) do not even mention Hazen or Williams in their *History of Hydraulics*. Another commonly used equation was developed by Scobey based on experiments in concrete pipes.

$$h = \left(\frac{0.00669}{C_s}\right)^2 \frac{Lv^2}{D^{1.25}}$$

Where C_s = Scobey's coefficient
The 0.00669 becomes 0.0105 in English units

In was the German "rocket scientists" in the early 20th century who developed a more thorough of the relationship between solid bodies and moving fluids. Ludwig Prandtl and his associates Theodor von Karmen, Johan Nikuradse Henrich Blasius and Thomas Stanton determined that the nature of the boundary layer between the fluid and solid phases determines drag (and head loss). Their work at the Kaiser Wilhelm Institute for Stromungforshung identified three types of pipe boundary layers: laminar, transition and turbulent

Nikuradse developed the famous experiments where uniform sand grains were glued inside of pipes and the head loss was measured for various velocities. These relationships are summarize in diagrams by Stanton, Rouse and later Moody showing the relationship between Reynolds number, pipe roughness and friction factor (Walski et al., 2003). Rouse (Brown, 2003) published a chart for estimating f in 1942 but f appeared on two axes and is somewhat difficult to use. Moody's diagram published in 1944 is easier to use and is more widely accepted.

The most commonly used equation for friction factor in pipes is the Colebrook-White equation which gives the relationship between pipe relative roughness, Reynolds number and friction factor as

$$\frac{1}{\sqrt{f}} = -0.86 \quad \ln\left(\frac{\varepsilon / D}{3.7} + \frac{2.51}{R\sqrt{f}}\right)$$

Where ε = equivalent sand grain roughness

The Colebrook-White equation cannot be solved explicitly for f, which led to the development of numerous explicit equations to approximate Colebrook-White. The most widely accepted is Swamee-Jain (1976).

$$f = \frac{1.325}{\left[\ln\left(\frac{\varepsilon}{3.7D} + \frac{5.75}{R^{0.9}}\right)\right]^2}$$

While there was a general awareness of hydraulic transients before his time, in 1897 Nicolai Joukowsky was the first to demonstrate both theoretically and experimentally the acoustic nature of waterhammer due to valve closure and present an equation that related the magnitude of transient pressure to the change in velocity (Rouse and Ince, 1980).

$$dh = \pm \frac{a}{g} dv$$

Where a = characteristic wave celerity in closed conduit

Joukowksy's work on transients was extended in the early 1900's by Lorenzo Allievi. Parmakien developed graphical methods for analyzing transients.

NETWORK SOLUTIONS

The equations presented above describe flow in individual closed conduits. However, real water distribution systems are made up of thousands of pipe elements. Solving for flows and pressures in a real water distribution system involves solving thousands of simultaneous nonlinear equations. Until recent advances in computer modeling, such calculations were impossible. Engineers' ability to design and construct systems exceeded the profession's ability to analyze them. Nevertheless, engineers through the early 20th century were able to design and analyze functioning water distribution system hydraulics using a combination of simplifications, rules-of-thumb and conservatism.

Freeman developed a graphical method for solving problems with parallel pipes in the late 1800s and equivalent pipe methods were used to decompose complex problems in the early 1900s (Ramalingam, Lingireddy and Ormsbee, 2002). The graphical methods involved preparing curves of flow vs. head loss and working quickly through the system such that head losses in series are cumulative and head loss in a loop is zero. Aldrich (1937) reported that the curves could be drawn on thin celluloid sheets and these sheets could be superimposed to facilitate calculations. Stanley (1937) reported on successful application of these graphical methods.

Nevertheless, looped systems required tedious iterative calculations with heavy use of slide rules. Hardy Cross (Cross, 1936) at the University of Illinois developed a systematic tabular process for calculating system hydraulics. Cross' method was a variation of a method he had developed for solving structural equations. The method is based upon iteratively correcting flows around a loop

$$\Delta Q = \frac{\sum rQ^n}{\sum \left| nrQ^{n-1} \right|}$$

Where r = pipe resistance coefficient
n = 2 for Darcy-Weisbach and 1.85 for Hazen-Williams

Richard Southwell from Oxford University and the Imperial College of Science and Technology in London, also developed numerical methods, which he called "systematic relaxation of restraints" for solving pipe network problems (Rouse and Ince, 1980).

While this codified the iterative procedures, the calculations still involved extensive slide rule use. Camp (1943) summarized the state-of-the-art of manual hydraulic analysis for networks and noted that better field data was more important than theoretical calculations. Ramalingam, Lingireddy and Ormsbee (2002) categorized these pre-computer methods as

- Freeman's Graphical Method
- The Equivalent Pipe Method
- The Electric Network Analyzer
- Hardy Cross Method (s)
- The Simultaneous Node Method
- The Simultaneous Loop Method
- The Linear Method (Simultaneous Pipe Method)
- The Gradient Method (Simultaneous Component Method)

Analysis of any pipe network with multiple tanks or a single loop involve numerical solutions because it is impossible to separate the energy and continuity equations as can be done in strictly branched flow systems. Before the coming of digital computers, an engineer's job involved tedious calculations. Changing a single parameter required starting the solution essentially from scratch. Engineers therefore tended to highly skeletonize pipe network problems so that they could be solved manually in a feasible period of time. This was successful because in most problems because only a handful of pipes are truly important in developing a solution to a particular situation even though there may be many thousands of pipes in the network. The concept of network skeletonization has carried over into computerized solutions.

Special hydraulic slide rules were developed with the appropriate head loss formulas hard coded into the slide rule. This simplified calculations with exponents such as 1.852. Other shortcuts such as replacing groups of pipes by equivalent pipes and replacing minor loss elements by equivalent pipe lengths helped reduce the numerical burden of solving pipe networks.

Use of rules of thumb such as: no pipe diameters less than 150 mm (6 in.) in systems providing fire protection and maximum velocities less than approximately 3 m/s (10 ft/s), narrowed down problems into ones that could be manually solved.

Other rules-of-thumb were used for the placement of fire hydrants and the ISO (insurance Services Office) formula was used for evaluating the capacity of existing systems.

$$Q_r = Q_t \left(\frac{P_s - P_r}{P_s - P_t} \right)^{0.54}$$

Where Q_r = flow at residual pressure r
Q_t = flow during fire flow test
P_s = static pressure
P_t = pressure during fire flow test
P_r = residual pressure at which flow is to be determined

The need for theoretical calculations of system flows was further reduced by the development, in the late 1900s, of a practical, in-pipe Pitot rod which enabled engineers to directly measure flow inside of pressure pipes while they were in service and produce a chart of flow vs. time.

The first computer solutions of network problems were done on analog computers where electrical elements are used to simulate pipe networks. The McIlroy Network Analyzer was used by utilities from the early 1950s through the early 1970s (Walski et al., 2003). The last analog computer model was located at Springfield College.

With the coming of digital computers, water distribution system analysis has become significantly more powerful because of the ability of modern computers to handle computations much more quickly than could be handled with manual calculations.

References

Aldrich, E.H., 1937, "Solution of transmission problems of a water system." Transactions of A.S.C.E., 1579-1619.

Brown, G.O., 2003, "The History of the Darcy-Weisbach Equation for Pipe Flow Resistance," ASCE 150 th Anniversary Conference.

Camp, T.R. 1943. "Hydraulics of distribution systems – Some recent development in methods of analysis." Journal of the New England Water Works Association, 334-368.

Cross, H. 1936, *Analysis of Flow in Networks of Conduits or Conductors,* Univ. of Illinois Experiment Station, Bulletin No. 286, Urbana, Ill.

Mays, L.W., 2000, *Water Distribution Systems Handbook,* McGraw-Hill, New York.

Ramalingam, D., S. Lingireddy and L. E. Ormsbee, 2002, "History of Water Distribution network Analysis: over 100 Years of Progress," *Environmental and Water Resources History Proceedings,* ASCE 150th Anniversary Conference, Washington, DC.

Rouse, H. and Ince, S. 1980, *History of Hydraulics,* Iowa Institute of Hydraulic Research, Iowa City, Iowa.

Sanks, R.L, 2005. "Water Transport", in "Water Storage, Transport, and Distribution," edited by Yutaka Takahasi in *Encyclopedia of Life Support Systems* (EOLSS). Developed under the auspices of the UNESCO, EOLSS Publishers, Oxford, UK.

Stanley, C.M. (1937). A discussion on "Solution of transmission problems of a water system" by Aldrich, E.H., Transactions of A.S.C.E., 1579-1619.

Williams, G.S. and A. Hazen, 1906, *Hydraulic Tables,* John Wiley & Sons, New York.